## 附录 1

附录 1 展示式（3）对数似然函数的推导过程。

对于 n 个观测，预测变量为 $\boldsymbol{X}_i$，$\boldsymbol{Z}_i$，生存时间为 $Y_i$，示性函数为 $\tau_i, i=1,2,\cdots,n,$ 本文考虑部分线性半参数 AFT 模型：$\log T_i = \boldsymbol{X}_i^T\boldsymbol{\beta} + h(\boldsymbol{Z}_i) + \varepsilon_i$。

记 $f(\cdot)$、$S(\cdot)$ 分别为生存时间的密度函数和生存函数，$f_\varepsilon(\cdot)$ 和 $S_\varepsilon(\cdot)$ 分别为误差项的密度函数和生存函数。对于 $n$ 个观测 $(Y_i, \tau_i, \boldsymbol{X}_i, \boldsymbol{Z}_i)$，似然函数为 $\prod_{i=1}^n [f(Y_i)]^{\tau_i}[S(Y_i)]^{1-\tau_i}$。

由于 $f(Y_i) = f_\varepsilon(\varepsilon_i)/Y_i$，$S(Y_i) = S_\varepsilon(\varepsilon_i)$，因此似然函数等价于 $\prod_{i=1}^n [f_\varepsilon(\varepsilon_i)/Y_i]^{\tau_i}[S_\varepsilon(\varepsilon_i)]^{1-\tau_i}$，则对数似然（Log Likelihood，以下简称 LL）函数为：

$$
\begin{aligned}
\mathrm{LL} &= \log\left(\prod_{i=1}^n [f_\varepsilon(\varepsilon_i)/Y_i]^{\tau_i}[S_\varepsilon(\varepsilon_i)]^{1-\tau_i}\right) \\
&= \sum_{i=1}^n \left[\tau_i \log f_\varepsilon(\varepsilon_i) + (1-\tau_i)\log S_\varepsilon(\varepsilon_i) - \tau_i \log Y_i\right] \\
&= \sum_{i=1}^n \left[\tau_i \log f_\varepsilon(Y_i - \boldsymbol{X}_i\boldsymbol{\beta} - h(\boldsymbol{Z}_i)) + (1-\tau_i)\log S_\varepsilon(Y_i - \boldsymbol{X}_i\boldsymbol{\beta} - h(\boldsymbol{Z}_i)) - \tau_i \log Y_i\right]
\end{aligned}
$$

即论文中式（3）。

## 附录 2

附录 2 展示式（6）到式（7）的推导过程。

第 $k$+1 次更新 $\boldsymbol{\beta}$，目标函数为式（7）：

$$
\begin{aligned}
\boldsymbol{\beta}^{(k+1)} = \arg\max_{\boldsymbol{\beta}} \Bigg\{ & \frac{1}{n}\left[\boldsymbol{\tau}^T(\log\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)}) - (\boldsymbol{1} + \boldsymbol{\tau}^T)\boldsymbol{A}(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}, \boldsymbol{\delta}^{(k)})\right] \\
& -\lambda_1\|\boldsymbol{\beta}\|_1 - \lambda_2\left\|\boldsymbol{\delta}^{(k)}\right\|_1 - \frac{1}{2}\lambda_3\boldsymbol{\alpha}^{(k)T}\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)}\Bigg\}
\end{aligned} \tag{6}
$$

式（6）是关于 $\boldsymbol{\beta}$ 的非线性函数，不易求解。受 Simon 等（2011）算法思想的启发，以 $(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta}^{(k)})$ 为中心，将似然函数 $l_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}, \boldsymbol{\delta}^{(k)})$ 二阶泰勒展开，下面给出具体的推导过程。

令 $l_n(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\delta}) = \boldsymbol{\tau}^T(\log\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{K}^{(g)}(\boldsymbol{\delta})\boldsymbol{\alpha}) - (1+\boldsymbol{\tau}^T)\boldsymbol{A}(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\delta})$，$\boldsymbol{\eta} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{K}^{(g)}(\boldsymbol{\delta})\boldsymbol{\alpha}$。记 $l_n'(\boldsymbol{\eta})$，$l_n''(\boldsymbol{\eta})$ 分别表示 $l_n(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\delta})$ 关于 $\boldsymbol{\eta}$ 的梯度和 Hessian 矩阵，设 $\dot{l}_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}, \boldsymbol{\delta}^{(k)})$，$\ddot{l}_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}, \boldsymbol{\delta}^{(k)})$ 分别表示 $l_n(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\delta})$ 关于 $\boldsymbol{\beta}$ 的梯度和Hessian 矩阵。令 $\tilde{\boldsymbol{\eta}} = \boldsymbol{X}\boldsymbol{\beta}^{(k)} + \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)}$，将 $l_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}, \boldsymbol{\delta}^{(k)})$ 在第 $k$ 步估计值 $(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta}^{(k)})$ 处二阶泰勒展开，则

$$
\begin{aligned}
& l_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}, \boldsymbol{\delta}^{(k)}) \\
& \approx l_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta}^{(k)}) + (\boldsymbol{\beta} - \boldsymbol{\beta}^{(k)})^T\dot{l}_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta}^{(k)}) + \frac{1}{2}(\boldsymbol{\beta} - \boldsymbol{\beta}^{(k)})^T\ddot{l}_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta}^{(k)})(\boldsymbol{\beta} - \boldsymbol{\beta}^{(k)}) \\
& = l_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta}^{(k)}) + (\boldsymbol{\beta} - \boldsymbol{\beta}^{(k)})^T\boldsymbol{X}^T l_n'(\tilde{\boldsymbol{\eta}}) + \frac{1}{2}(\boldsymbol{\beta} - \boldsymbol{\beta}^{(k)})^T\boldsymbol{X}^T l_n''(\tilde{\boldsymbol{\eta}})\boldsymbol{X}(\boldsymbol{\beta} - \boldsymbol{\beta}^{(k)}) \\
& = l_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta}^{(k)}) + (\boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{X}\boldsymbol{\beta}^{(k)})^T l_n'(\tilde{\boldsymbol{\eta}}) + \frac{1}{2}(\boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{X}\boldsymbol{\beta}^{(k)})^T l_n''(\tilde{\boldsymbol{\eta}})(\boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{X}\boldsymbol{\beta}^{(k)}) \\
& = l_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta}^{(k)}) + (\boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)} - \boldsymbol{X}\boldsymbol{\beta}^{(k)} - \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)})^T l_n'(\tilde{\boldsymbol{\eta}}) \\
& \quad + \frac{1}{2}(\boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)} - \boldsymbol{X}\boldsymbol{\beta}^{(k)} - \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)})^T l_n''(\tilde{\boldsymbol{\eta}})(\boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)} - \boldsymbol{X}\boldsymbol{\beta}^{(k)} - \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)}) \\
& = l_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta}^{(k)}) + (\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n'(\tilde{\boldsymbol{\eta}}) + \frac{1}{2}(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n''(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}}) \\
& = (\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n'(\tilde{\boldsymbol{\eta}}) + \frac{1}{2}(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n''(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}}) + C_1^{(k)}
\end{aligned}
$$

$$=\frac{1}{2}\left\{(\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}})^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}})+2(\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}})^T l_n^{'}(\tilde{\boldsymbol{\eta}})\right\}+C_1^{(k)}$$

$$=\frac{1}{2}\left\{(\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}})^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}})+(\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}})^T l_n^{''}(\tilde{\boldsymbol{\eta}})l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})+[l_n^{'}(\tilde{\boldsymbol{\eta}})]^T[l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}]^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}})\right\}+C_1^{(k)}$$

$$=\frac{1}{2}\left\{(\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}})^T l_n^{''}(\tilde{\boldsymbol{\eta}})[\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}}+l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})]+[l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})]^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}})\right\}+C_1^{(k)}$$

$$=\frac{1}{2}\left\{(\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}})^T l_n^{''}(\tilde{\boldsymbol{\eta}})[\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}}+l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})]+[l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})]^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}})\right\}+\frac{1}{2}[l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})]^T l_n^{''}(\tilde{\boldsymbol{\eta}})l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})+C_2^{(k)}$$

$$=\frac{1}{2}[\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}}+l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})]^T l_n^{''}(\tilde{\boldsymbol{\eta}})[\boldsymbol{\eta}-\tilde{\boldsymbol{\eta}}+l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})]+C_2^{(k)}$$

$$=\frac{1}{2}[\tilde{\boldsymbol{\eta}}-l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})-\boldsymbol{\eta}]^T l_n^{''}(\tilde{\boldsymbol{\eta}})[\tilde{\boldsymbol{\eta}}-l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})-\boldsymbol{\eta}]+C_2^{(k)}$$

其中，$C_1^{(k)}, C_2^{(k)}$ 是与参数 $\boldsymbol{\beta}$ 无关的项，其具体形式分别为 $C_1^{(k)}=l_n(\boldsymbol{\alpha}^{(k)},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})$，$C_2^{(k)}=-\frac{1}{2}[l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})]^T l_n^{''}(\tilde{\boldsymbol{\eta}})l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})+l_n(\boldsymbol{\alpha}^{(k)},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})$。

因此，$l_n(\boldsymbol{\alpha}^{(k)},\boldsymbol{\beta},\boldsymbol{\delta}^{(k)})\approx\frac{1}{2}[\tilde{\boldsymbol{\eta}}-l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})-\boldsymbol{\eta}]^T l_n^{''}(\tilde{\boldsymbol{\eta}})[\tilde{\boldsymbol{\eta}}-l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})-\boldsymbol{\eta}]+C_2^{(k)}$。易得极大化 $l_n(\boldsymbol{\alpha}^{(k)},\boldsymbol{\beta},\boldsymbol{\delta}^{(k)})$ 等价于极大化 $\frac{1}{2}[\tilde{\boldsymbol{\eta}}-l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})-\boldsymbol{\eta}]^T l_n^{''}(\tilde{\boldsymbol{\eta}})[\tilde{\boldsymbol{\eta}}-l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})-\boldsymbol{\eta}]$。

事实上，$l_n^{''}(\tilde{\boldsymbol{\eta}})$ 是一个对角矩阵，所以只需要计算 O($n$)项。令 $V(\tilde{\boldsymbol{\eta}})=\tilde{\boldsymbol{\eta}}-l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}l_n^{'}(\tilde{\boldsymbol{\eta}})$，$\boldsymbol{W}(\tilde{\boldsymbol{\eta}})$ 表示 $-l_n^{''}(\tilde{\boldsymbol{\eta}})$ 矩阵，$\boldsymbol{\beta}$ 的更新估计可通过求解以下惩罚加权最小二乘问题得到：

$$\frac{1}{2n}\left[V(\tilde{\boldsymbol{\eta}})-\boldsymbol{X}\boldsymbol{\beta}-\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)}\right]^T W(\tilde{\boldsymbol{\eta}})\left[V(\tilde{\boldsymbol{\eta}})-\boldsymbol{X}\boldsymbol{\beta}-\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)}\right]+\lambda_1\|\boldsymbol{\beta}\|_1 \tag{7}$$

以上是式（6）到式（7）的推导过程。

## 附录 3

附录 3 展示第 $k$+1 次更新 $\boldsymbol{\alpha}$ 和 $\boldsymbol{\delta}$ 的过程。

第 $k$+1 次更新 $\boldsymbol{\alpha}$，目标函数为：

$$\boldsymbol{\alpha}^{(k+1)}=\arg\max_{\boldsymbol{\alpha}}\left\{\frac{1}{n}\left[\boldsymbol{\tau}^T(\log\boldsymbol{Y}-\boldsymbol{X}\boldsymbol{\beta}^{(k)}-\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha})-(\boldsymbol{1}+\boldsymbol{\tau}^T)\boldsymbol{A}(\boldsymbol{\alpha},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})\right]\right.$$
$$\left.-\lambda_1\left\|\boldsymbol{\beta}^{(k)}\right\|_1-\lambda_2\left\|\boldsymbol{\delta}^{(k)}\right\|_1-\frac{1}{2}\lambda_3\boldsymbol{\alpha}^T\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}\right\}$$

上式是关于 $\boldsymbol{\alpha}$ 的非线性函数，不易直接求解。因此将 $l_n(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\delta})=\boldsymbol{\tau}^T(\log\boldsymbol{Y}-\boldsymbol{X}\boldsymbol{\beta}-\boldsymbol{K}^{(g)}(\boldsymbol{\delta})\boldsymbol{\alpha})-(1+\boldsymbol{\tau}^T)\boldsymbol{A}(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\delta})$ 在 $(\boldsymbol{\alpha}^{(k)},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})$ 处二阶泰勒展开，具体推导如下。

令 $\boldsymbol{\eta}=\boldsymbol{X}\boldsymbol{\beta}+\boldsymbol{K}^{(g)}(\boldsymbol{\delta})\boldsymbol{\alpha}$。记 $l_n^{'}(\boldsymbol{\eta})$，$l_n^{''}(\boldsymbol{\eta})$ 分别表示 $l_n(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\delta})$ 关于 $\boldsymbol{\eta}$ 的梯度和 Hessian 矩阵，$\dot{l}_n(\boldsymbol{\alpha},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})$，$\ddot{l}_n(\boldsymbol{\alpha},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})$ 分别表示 $l_n(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\delta})$ 关于 $\boldsymbol{\alpha}$ 的梯度和 Hessian 矩阵。令 $\tilde{\boldsymbol{\eta}}=\boldsymbol{X}\boldsymbol{\beta}^{(k)}+\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)}$，在第 $k$ 步估计值 $(\boldsymbol{\alpha}^{(k)},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})$ 处，将 $l_n(\boldsymbol{\alpha},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})$ 二阶泰勒展开，得

$$l_n(\boldsymbol{\alpha},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})$$

$$\approx l_n(\boldsymbol{\alpha}^{(k)},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})+(\boldsymbol{\alpha}-\boldsymbol{\alpha}^{(k)})^T\dot{l}_n(\boldsymbol{\alpha}^{(k)},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})+\frac{1}{2}(\boldsymbol{\alpha}-\boldsymbol{\alpha}^{(k)})^T\ddot{l}_n(\boldsymbol{\alpha}^{(k)},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})(\boldsymbol{\alpha}-\boldsymbol{\alpha}^{(k)})$$

$$=l_n(\boldsymbol{\alpha}^{(k)},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})+(\boldsymbol{\alpha}-\boldsymbol{\alpha}^{(k)})^T\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})^T l_n^{'}(\tilde{\boldsymbol{\eta}})+\frac{1}{2}(\boldsymbol{\alpha}-\boldsymbol{\alpha}^{(k)})^T\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})^T l_n^{''}(\tilde{\boldsymbol{\eta}})\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})(\boldsymbol{\alpha}-\boldsymbol{\alpha}^{(k)})$$

$$=l_n(\boldsymbol{\alpha}^{(k)},\boldsymbol{\beta}^{(k)},\boldsymbol{\delta}^{(k)})+(\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}-\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)})^T l_n^{'}(\tilde{\boldsymbol{\eta}})$$
$$+\frac{1}{2}(\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}-\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)})^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}-\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)})$$

$$= l_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta}^{(k)}) + (\boldsymbol{X}\boldsymbol{\beta}^{(k)} + \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha} - \boldsymbol{X}\boldsymbol{\beta}^{(k)} - \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)})^T l_n^{'}(\tilde{\boldsymbol{\eta}})$$

$$+\frac{1}{2}(\boldsymbol{X}\boldsymbol{\beta}^{(k)} + \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha} - \boldsymbol{X}\boldsymbol{\beta}^{(k)} - \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)})^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{X}\boldsymbol{\beta}^{(k)} + \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha} - \boldsymbol{X}\boldsymbol{\beta}^{(k)} - \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}^{(k)})$$

$$= l_n(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta}^{(k)}) + (\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n^{'}(\tilde{\boldsymbol{\eta}}) + \frac{1}{2}(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})$$

$$= (\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n^{'}(\tilde{\boldsymbol{\eta}}) + \frac{1}{2}(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}}) + C_1^{(k)}$$

$$= \frac{1}{2}\left\{(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}}) + 2(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n^{'}(\tilde{\boldsymbol{\eta}})\right\} + C_1^{(k)}$$

$$= \frac{1}{2}\left\{(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}}) + (\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n^{''}(\tilde{\boldsymbol{\eta}}) l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}}) + [l_n^{'}(\tilde{\boldsymbol{\eta}})]^T [l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1}]^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})\right\} + C_1^{(k)}$$

$$= \frac{1}{2}\left\{(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n^{''}(\tilde{\boldsymbol{\eta}})[\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}} + l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}})] + [l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}})]^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})\right\} + C_1^{(k)}$$

$$= \frac{1}{2}\left\{(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})^T l_n^{''}(\tilde{\boldsymbol{\eta}})[\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}} + l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}})] + [l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}})]^T l_n^{''}(\tilde{\boldsymbol{\eta}})(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}})\right\}$$

$$+\frac{1}{2}[l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}})]^T l_n^{''}(\tilde{\boldsymbol{\eta}}) l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}}) + C_2^{(k)}$$

$$= \frac{1}{2}[\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}} + l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}})]^T l_n^{''}(\tilde{\boldsymbol{\eta}})[\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}} + l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}})] + C_2^{(k)}$$

$$= \frac{1}{2}[\tilde{\boldsymbol{\eta}} - l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}}) - \boldsymbol{\eta}]^T l_n^{''}(\tilde{\boldsymbol{\eta}})[\tilde{\boldsymbol{\eta}} - l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}}) - \boldsymbol{\eta}] + C_2^{(k)}$$

其中，$C_1^{(k)}$，$C_2^{(k)}$ 是与参数 $\boldsymbol{\alpha}$ 无关的项，因此，

$$l_n(\boldsymbol{\alpha}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta}^{(k)}) \approx \frac{1}{2}[\tilde{\boldsymbol{\eta}} - l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}}) - \boldsymbol{\eta}]^T l_n^{''}(\tilde{\boldsymbol{\eta}})[\tilde{\boldsymbol{\eta}} - l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}}) - \boldsymbol{\eta}] + C_2^{(k)}$$

于是，极大化 $l_n(\boldsymbol{\alpha}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta}^{(k)})$ 等价于极大化下式：

$$\frac{1}{2}[\tilde{\boldsymbol{\eta}} - l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}}) - \boldsymbol{\eta}]^T l_n^{''}(\tilde{\boldsymbol{\eta}})[\tilde{\boldsymbol{\eta}} - l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}}) - \boldsymbol{\eta}]$$

不妨统一将极大化问题转化为极小化问题，令 $V(\tilde{\boldsymbol{\eta}})=\tilde{\boldsymbol{\eta}} - l_n^{''}(\tilde{\boldsymbol{\eta}})^{-1} l_n^{'}(\tilde{\boldsymbol{\eta}})$，$\boldsymbol{W}(\tilde{\boldsymbol{\eta}})$ 表示 $-l_n^{''}(\tilde{\boldsymbol{\eta}})$ 矩阵。则上式等价于最小化下式：

$$\frac{1}{2n}\left[V(\tilde{\boldsymbol{\eta}}) - \boldsymbol{X}\boldsymbol{\beta}^{(k)} - \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}\right]^T \boldsymbol{W}(\tilde{\boldsymbol{\eta}})\left[V(\tilde{\boldsymbol{\eta}}) - \boldsymbol{X}\boldsymbol{\beta}^{(k)} - \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}\right] + \frac{1}{2}\lambda_3 \boldsymbol{\alpha}^T \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}$$

可以看出，上式是一个关于 $\boldsymbol{\alpha}$ 的二次型。关于 $\boldsymbol{\alpha}$ 求导并令导数为零，有

$$\frac{1}{n}(-\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)}))\boldsymbol{W}(\tilde{\boldsymbol{\eta}})\left[V(\tilde{\boldsymbol{\eta}}) - \boldsymbol{X}\boldsymbol{\beta}^{(k)} - \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha}\right] + \lambda_3 \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{\alpha} = 0$$

即 $\left[\frac{1}{n}\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{W}(\tilde{\boldsymbol{\eta}})\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)}) + \lambda_3 \boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\right]\boldsymbol{\alpha} = \frac{1}{n}\boldsymbol{K}^{(g)}(\boldsymbol{\delta}^{(k)})\boldsymbol{W}(\tilde{\boldsymbol{\eta}})(V(\tilde{\boldsymbol{\eta}}) - \boldsymbol{X}\boldsymbol{\beta}^{(k)})$

求解以上线性方程组可得到 $\boldsymbol{\alpha}$ 的更新估计①。

第 $k$+1 次更新 $\boldsymbol{\delta}$：固定 $\boldsymbol{\alpha} = \boldsymbol{\alpha}^{(k)}$，$\boldsymbol{\beta} = \boldsymbol{\beta}^{(k)}$，最大化 $f(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta})$。受 Friedman 等（2010）算法思想启发，为实现多元参数 $\boldsymbol{\delta}$ 的有效更新，本文基于逐维梯度下降算法，循环更新 $\delta_l$，对于 $\delta_l > 0, \delta_{l'} > 0, l, l' = 1, \cdots, q$，有

$$\delta_q^{(k+1)} = \arg\max_{\delta_q}\left\{\frac{1}{n}\left[\boldsymbol{\tau}^T(\log \boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta}^{(k)} - \boldsymbol{K}^{(g)}(\boldsymbol{\delta})\boldsymbol{\alpha}^{(k)}) - (\boldsymbol{1} + \boldsymbol{\tau}^T)\boldsymbol{A}(\boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\delta})\right]\right.$$

①若上述方程的左侧为奇异矩阵，线性方程组求解会失效，因此不妨在方程组左侧添加对角元素为的对角矩阵以修正线性方程组。

$$-\lambda_2 \sum_{l^{\cdot} \neq l}^{q} \delta_{l^{\cdot}}^{(k)} - \lambda_2 \delta_l - \frac{1}{2} \lambda_3 \boldsymbol{\alpha}^{(k)T} \boldsymbol{K}^{(g)}(\boldsymbol{\delta}) \boldsymbol{\alpha}^{(k)} \Big\}$$

上式是关于 $\delta_l$ 的带限制条件的一元非线性最优化问题，可利用 R 软件中的最优化算法，如 L-BFGS-B，寻找未知参数的最优解。

## 附录 4

C-statistic：C-statistic表示在所有“可用对”中预测结果和观察结果相一致的概率。其范围在0.5~1之间，其值越高表明模型的预测能力越好。对于预先指定的随访期 $(0, t_0)$，定义C-statistic为 $C_{t_0} = P(f(\boldsymbol{X}_1, \boldsymbol{Z}_1) < f(\boldsymbol{X}_2, \boldsymbol{Z}_2) \mid Y_1 < Y_2, Y_1 < t_0)$，其中 $Y$ 表示观察的生存时间，$f(\boldsymbol{X}, \boldsymbol{Z})$ 表示估计的风险得分。特别地，对于RegGKM方法，$f(\boldsymbol{X}, \boldsymbol{Z}) = \boldsymbol{X}^T \hat{\boldsymbol{\beta}} + \hat{h}(\boldsymbol{Z})$。基于“逆概率加权”技术，有

$$\hat{C}_{t_0} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \tau_i \{\hat{G}(Y_i)\}^{-2} \mathrm{I}(Y_i < Y_j, Y_i < t_0) \mathrm{I}(f(\boldsymbol{X}_i, \boldsymbol{Z}_i) < f(\boldsymbol{X}_j, \boldsymbol{Z}_j))}{\sum_{i=1}^{n} \sum_{j=1}^{n} \tau_i \{\hat{G}(Y_i)\}^{-2} \mathrm{I}(Y_i < Y_j, Y_i < t_0)}$$

其中 $\mathrm{I}(\cdot)$ 是示性函数，$\hat{G}(\cdot)$ 是删失时间分布 $G(t) = P(T > t)$ 的Kaplan-Meier估计量。本文利用R包“survC1”实现C-statistic的计算。

当模型中包含冗余预测变量时，本文借助于Error（E）、correct-selection（C）、over-selection（O）、under-selection（U）四个指标[①]比较RegGKM方法和LASSO方法的变量选择效果（Rong等，2018）。若C和O越高，E和U越低，表明将无关预测变量选入模型的频率更低，变量选择效果更好。

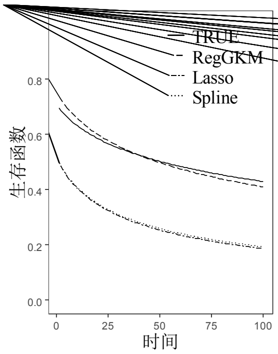
附表 1　RegGKM 和 LASSO 方法的变量选择

| 设置 | CR | 方法 | Z | | | | X | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | E | C | O | U | E | C | O | U |
| 2 | 0% | RegGKM | 0.53 | 0.27 | 0.73 | 0.00 | 0.87 | 0.07 | 0.83 | 0.10 |
| | | LASSO | 0.89 | 0.00 | 0.66 | 0.34 | 0.90 | 0.10 | 0.90 | 0.00 |
| | 10% | RegGKM | 0.48 | 0.18 | 0.82 | 0.00 | 0.82 | 0.06 | 0.82 | 0.12 |
| | | LASSO | 0.88 | 0.00 | 0.74 | 0.26 | 0.85 | 0.15 | 0.85 | 0.00 |
| | 30% | RegGKM | 0.52 | 0.22 | 0.76 | 0.02 | 0.88 | 0.08 | 0.85 | 0.07 |
| | | LASSO | 0.92 | 0.00 | 0.73 | 0.27 | 0.93 | 0.07 | 0.92 | 0.01 |
| | 50% | RegGKM | 0.57 | 0.12 | 0.84 | 0.04 | 0.90 | 0.09 | 0.85 | 0.06 |
| | | LASSO | 0.92 | 0.00 | 0.72 | 0.28 | 0.95 | 0.04 | 0.95 | 0.01 |
| 3 | 0% | RegGKM | 0.27 | 0.04 | 0.96 | 0.00 | 0.72 | 0.12 | 0.72 | 0.16 |
| | | LASSO | 0.91 | 0.00 | 0.68 | 0.32 | 0.91 | 0.04 | 0.96 | 0.00 |
| | 10% | RegGKM | 0.33 | 0.05 | 0.95 | 0.00 | 0.66 | 0.11 | 0.67 | 0.22 |
| | | LASSO | 0.86 | 0.00 | 0.52 | 0.48 | 1.00 | 0.00 | 1.00 | 0.00 |
| | 30% | RegGKM | 0.36 | 0.11 | 0.88 | 0.01 | 0.72 | 0.16 | 0.72 | 0.12 |
| | | LASSO | 0.82 | 0.00 | 0.61 | 0.39 | 0.83 | 0.17 | 0.83 | 0.00 |
| | 50% | RegGKM | 0.52 | 0.06 | 0.94 | 0.00 | 0.78 | 0.15 | 0.79 | 0.06 |
| | | LASSO | 0.92 | 0.00 | 0.71 | 0.29 | 0.92 | 0.07 | 0.93 | 0.00 |
| 4 | 0% | RegGKM | 0.32 | 0.20 | 0.77 | 0.03 | 0.84 | 0.08 | 0.78 | 0.14 |
| | | LASSO | 0.87 | 0.00 | 0.59 | 0.41 | 0.88 | 0.00 | 1.00 | 0.00 |
| | 10% | RegGKM | 0.42 | 0.20 | 0.75 | 0.05 | 0.87 | 0.20 | 0.60 | 0.20 |
| | | LASSO | 0.87 | 0.00 | 0.55 | 0.45 | 0.88 | 0.00 | 1.00 | 0.00 |
| | 30% | RegGKM | 0.37 | 0.12 | 0.82 | 0.06 | 0.89 | 0.17 | 0.71 | 0.12 |
| | | LASSO | 0.88 | 0.00 | 0.70 | 0.30 | 0.91 | 0.00 | 0.99 | 0.01 |

[①] E 表示选入模型的冗余变量占全部冗余变量的频率，C 表示只选出全部相关预测变量的频率，O 表示选出全部相关和部分冗余预测变量的频率，U 表示未选出全部相关预测变量的频率。

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 50% | RegGKM | 0.34 | 0.14 | 0.73 | 0.13 | 0.90 | 0.10 | 0.76 | 0.14 |
| | | LASSO | 0.89 | 0.00 | 0.66 | 0.34 | 0.91 | 0.00 | 0.91 | 0.09 |
| 5 | 0% | RegGKM | 0.28 | 0.20 | 0.78 | 0.02 | 0.81 | 0.09 | 0.79 | 0.12 |
| | | LASSO | 0.86 | 0.00 | 0.57 | 0.43 | 0.86 | 0.00 | 1.00 | 0.00 |
| | 10% | RegGKM | 0.30 | 0.16 | 0.83 | 0.01 | 0.87 | 0.10 | 0.79 | 0.11 |
| | | LASSO | 0.87 | 0.00 | 0.65 | 0.35 | 0.87 | 0.00 | 1.00 | 0.00 |
| | 30% | RegGKM | 0.27 | 0.09 | 0.80 | 0.11 | 0.88 | 0.11 | 0.77 | 0.12 |
| | | LASSO | 0.85 | 0.00 | 0.62 | 0.38 | 0.89 | 0.00 | 1.00 | 0.00 |
| | 50% | RegGKM | 0.76 | 0.23 | 0.58 | 0.19 | 0.88 | 0.08 | 0.81 | 0.11 |
| | | LASSO | 0.89 | 0.00 | 0.65 | 0.35 | 0.90 | 0.00 | 0.96 | 0.04 |
| 6 | 0% | RegGKM | 0.52 | 0.02 | 0.98 | 0.00 | -- | -- | -- | -- |
| | | LASSO | 0.98 | 0.00 | 0.92 | 0.08 | -- | -- | -- | -- |
| | 10% | RegGKM | 0.53 | 0.04 | 0.96 | 0.00 | -- | -- | -- | -- |
| | | LASSO | 0.99 | 0.00 | 0.98 | 0.02 | -- | -- | -- | -- |
| | 30% | RegGKM | 0.47 | 0.03 | 0.97 | 0.00 | -- | -- | -- | -- |
| | | LASSO | 0.96 | 0.00 | 0.90 | 0.10 | -- | -- | -- | -- |
| | 50% | RegGKM | 0.42 | 0.03 | 0.95 | 0.02 | -- | -- | -- | -- |
| | | LASSO | 0.93 | 0.00 | 0.80 | 0.20 | -- | -- | -- | -- |
| 7 | 0% | RegGKM | 0.52 | 0.02 | 0.96 | 0.02 | 0.65 | 0.02 | 0.86 | 0.12 |
| | | LASSO | 0.98 | 0.00 | 0.94 | 0.06 | 0.85 | 0.00 | 0.98 | 0.02 |
| | 10% | RegGKM | 0.49 | 0.01 | 0.97 | 0.02 | 0.74 | 0.01 | 0.90 | 0.09 |
| | | LASSO | 0.98 | 0.00 | 0.98 | 0.02 | 0.83 | 0.00 | 0.94 | 0.06 |
| | 30% | RegGKM | 0.46 | 0.02 | 0.96 | 0.02 | 0.73 | 0.02 | 0.88 | 0.10 |
| | | LASSO | 0.96 | 0.00 | 0.92 | 0.08 | 0.85 | 0.02 | 0.92 | 0.06 |
| | 50% | RegGKM | 0.42 | 0.01 | 0.95 | 0.04 | 0.72 | 0.01 | 0.91 | 0.08 |
| | | LASSO | 0.96 | 0.00 | 0.90 | 0.10 | 0.82 | 0.01 | 0.88 | 0.10 |

注：“--”表示无变量选择结果，设置 6 中 $\boldsymbol{X}$ 不包含冗余变量，因此没有变量选择结果。由于 Spline 方法只可得到线性 AFT 模型的参数估计，无法实现变量选择，因此附表 1 中不涉及该方法。

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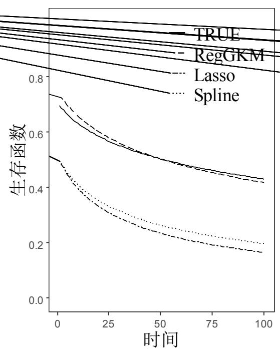


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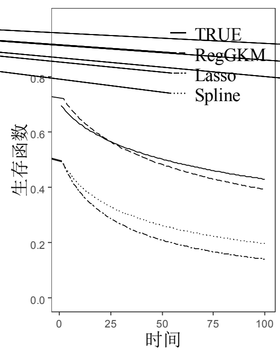


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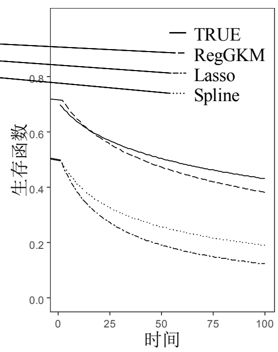


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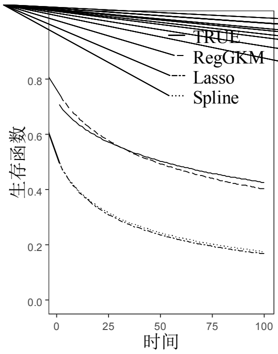


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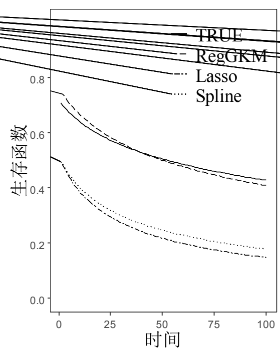


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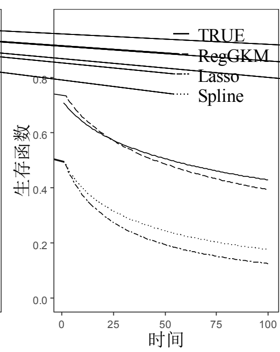


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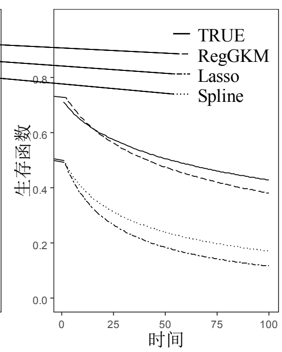


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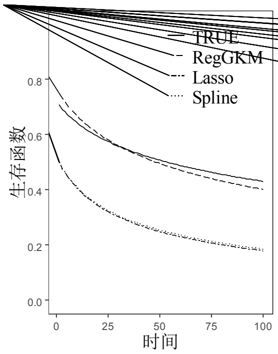


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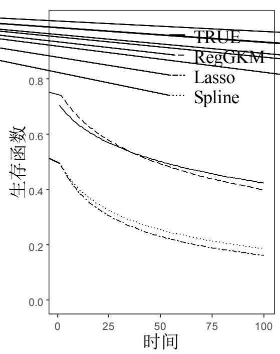


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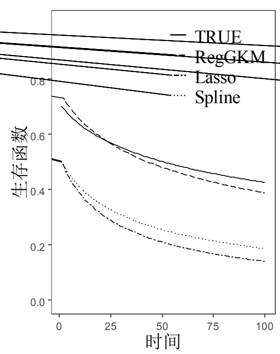


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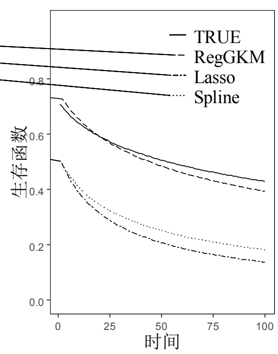


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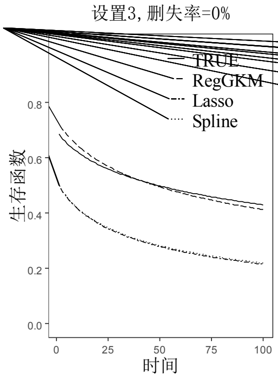


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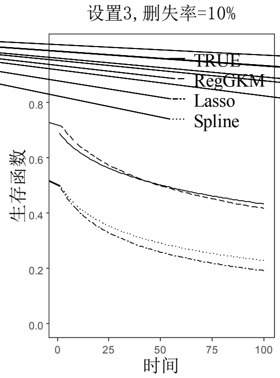


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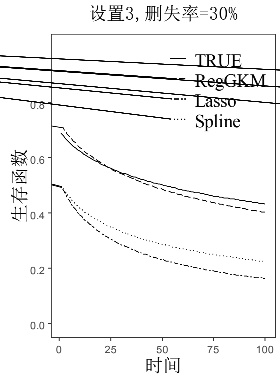


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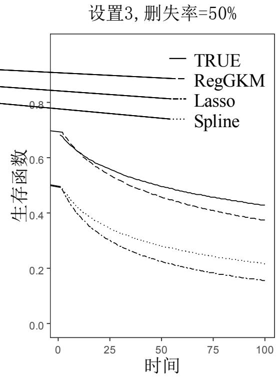


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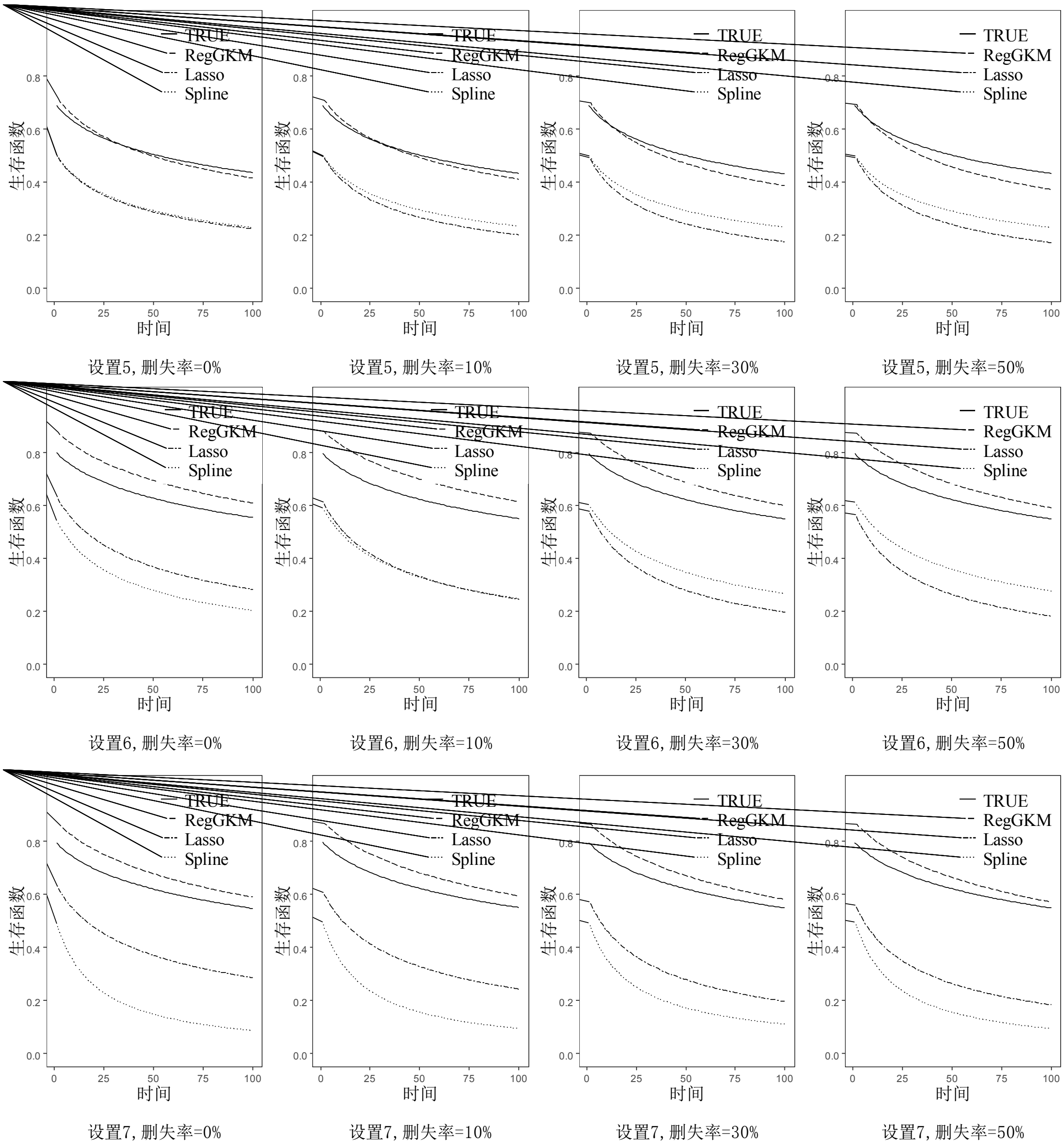


附图 1　RegGKM、LASSO 和 Spline 方法的生存函数估计图

注：黑色实线表示真实的生存函数曲线（TRUE），虚线、双划线、点划线分别表示 RegGKM、LASSO 和 Spline 方法的生存函数估计。